Urban unemployment, privatization policy, and a differentiated mixed oligopoly

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Abstract

The purpose of this paper is to analyze the effect of privatization of public firm on urban unemployment in a differentiated mixed oligopoly. This model is extended by combining Harris and Todaro model with a differentiated mixed oligopoly. We introduce the quasi-linear utility function with a quadratic sub-utility to derive the solution of model analytically. As the results of our analysis, the urban unemployment depends on the privatization level of public firm and the progress of privatization of public firm worsen it when the intensity of preferences for differentiated products is large relatively. As for social welfare, we show that the privatization of public firm improves social welfare under certain conditions.

Key words: dualistic economy, urban-rural migration, mixed oligopoly, product differentiation
JEL: J62, O18, L32,

1 Introduction

This paper combines a traditional dualistic economy model with a differentiated mixed oligopoly model. In development economics, Harris and Todaro[6], which is one of pioneer studies in dualistic economy, describes a dualistic economy by assuming the downward rigidity of wage in urban area and explains

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the occurrence of unemployment endogenously. Though there is no room for doubt that Harris and Todaro model is one of most important studies about dualistic economy analysis, it is hard to say that the setting necessarily accords with reality. Harris and Todaro[6] has been extended from the various viewpoints. Corden and Findlay[2] extend Harris and Todaro model by taking account of mobile capital between regions. Calvo[1] introduces the behavior of labor unions into Harris and Todaro model and determines the higher fixed wage in urban areas endogenously. Fukuyama and Naito[5] introduce polluting goods into Harris and Todaro model and analyze the effect of environmental policy on urban unemployment. Naito[9] introduces a mixed duopoly into Fukuyama and Naito[5] and analyzes the effect of privatization of public firm on urban employment. However, Naito[9] deals with not differentiated goods but homogeneous goods. In the actual world, it is not natural to assume the homogeneous goods produced in urban area.

As for a mixed oligopoly, De Fraja and Delbono[3] construct a mixed oligopoly market, where private firms compete with a public firm in the homogeneous goods market. As for a mixed oligopoly, De Fraja and Delbono[3] construct a mixed oligopoly market, where private firms compete with a public firm in the homogeneous goods market. De Fraja and Delbono[3] compares the mixed oligopoly, which a public firm is full nationalized, with the pure oligopoly, which a public firm is full privatization. Matsumura[7] considers the model where the public firm maximizes the weighted average of social welfare and profit as the objective function and show that the partial privatization of public firm is optimal. Matsumura and Kanda[8] take account of allow free entry of private firms in the mixed oligopoly. Fujiwara[4] consider the quasi-linear utility function constructed by Ottaviano, Tabuchi, and Thisse [11] and introduces product differentiation into a mixed oligopoly model. However, Fujiwara[4] consider the quasi-linear utility function constructed by Ottaviano, Tabuchi, and Thisse[11] and introduces product differentiation into a mixed oligopoly model. Though Fujiwara[4] adopts the quasi-linear utility function of Ottaviano, Tabuchi, and Thisse[11] in his paper, he does not take account of monopolistic competition.

As for the share of the public firm, the areas with share of less than 10% increases from 2003 to 2009 though the trend of difference among regions does not change between 2003 and 2009. Next we refer to urban employment rate in China. The urban unemployment in urban area of China was 4.09 % in 2010 though the average unemployment rate in China was 2.95%. Thus, it is possible that the migration occurs from rural area to urban area.

As is generally known, the the recent economic development of China is remarkable. However, there is a difference in the level of economic development there. Figure 1 and Figure 2 describe the share of public firms of each region in China, respectively. As we know from those figures, we know that the share

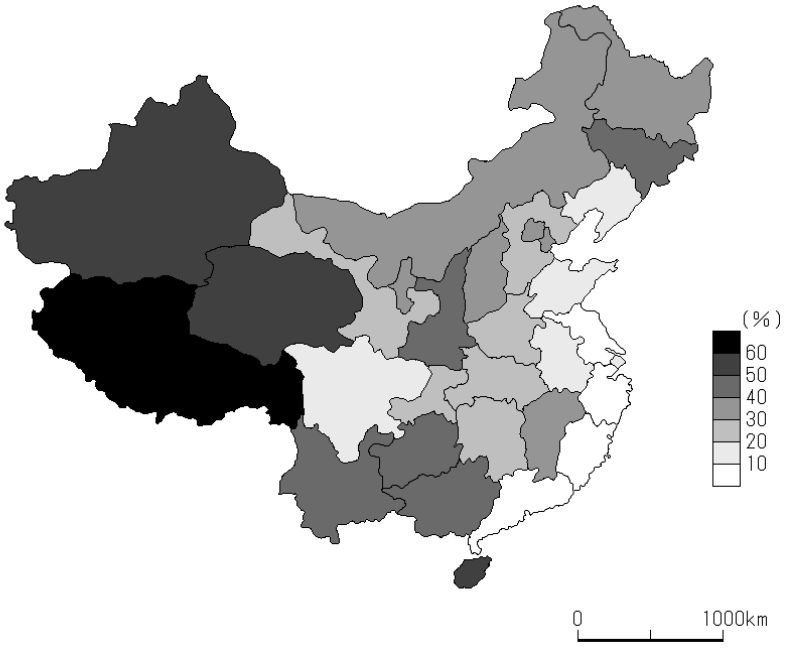

Fig. 1. The share of public firm in China (2003) **Source**: China Statistical Yearbook 2004

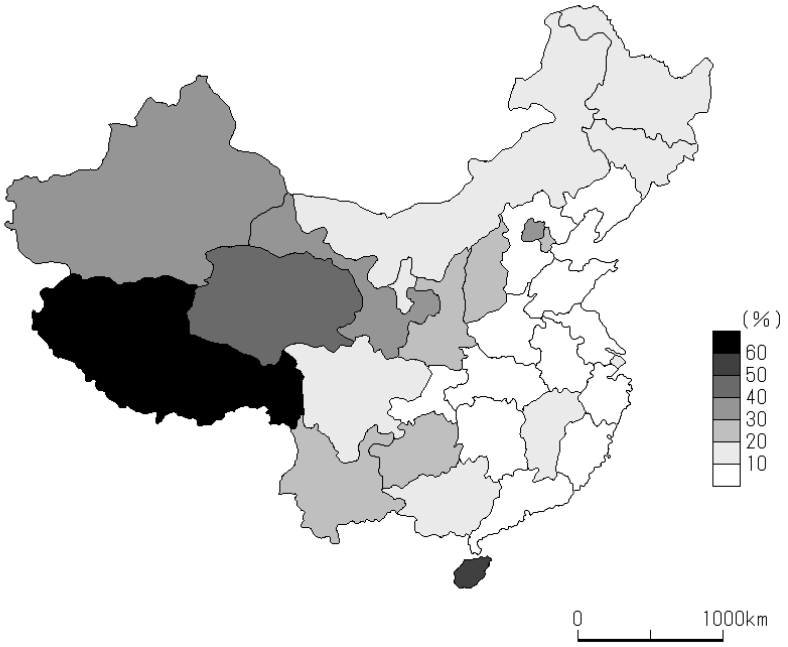

Fig. 2. The share of public firm in China (2009) **Source**: China Statistical Yearbook 2010

of public firms are different in each region of China. As for the share of the public firm, the areas with share of less than 10% increases from 2003 to 2009 though the trend of difference among regions does not change between 2003

and 2009. The data of the unemployment rate in China do not necessarily evaluate reality by the definition. Because the unemployment rate in China is the unemployment rate for households with register registration was accomplished, it is possible that this statistical value may be underestimated. Since it is thought that the privatization of the public firm brings any influence for labor demand through the optimization of public firm, it is necessary for the government to consider the privatization of public firm by taking account of urban unemployment.

Thus, we combine a traditional dualistic economy model with a differentiated mixed oligopoly model to construct the model which enable to analyze the urban unemployment and the privatization of public firm of privatization under a differentiated mixed oligopoly. Thus, we combine a traditional dualistic economy model with a differentiated mixed oligopoly model to construct the model, which enables to analyze the urban unemployment and the privatization of public firm of privatization under a differentiated mixed oligopoly. We adopt Ottaviano, Tabuchi, and Thisse[11] and Fujiwara[4] to describe a product differentiation and a mixed oligopoly market. On the other hand, we use Harris and Todaro[6] to describe a traditional dualistic economy. We consider the effect of privatization of public firm on urban unemployment or social welfare in this model.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the properties of market equilibrium. Based on section 2, section 3 clarifies the effects of public firm privatization on migration between urban area and rural area, urban unemployment, and social welfare. The last section presents concluding remarks.

2 The model

We consider the economy consists of two regions in our model. One of them is urban area, the other is rural area. Following Harris and Todaro[6], every household in the economy can migrate between regions without any cost. We consider the three types of household in this model. Indexes c, r, and u denote the household employed by the manufactured goods sector, those employed by agricultural goods sector, and those who are not employed in both sectors and reside in urban area, respectively.

Now we consider two kinds of products: agricultural goods and manufactured goods. The manufactured goods and agricultural goods are produced in urban area and rural area, respectively. Though manufactured goods sector require labor and capital as input factors to produce their own products, the agricultural goods sector requires only labor as input factor. Though each man-

ufactured goods are differentiated and are produced by one public firms and n-private firms in urban area, the agricultural goods are homogeneous.

2.1 Household

We assume that the urban wage is fixed above the market-clearing level of labor market, and that it has downward rigidity attributable to the minimum wage system and so on. Particularly, let $\bar{w}(= w_c)$ represent the minimum wage in the urban area, which is higher than the rural wage w_r. That rural wage is determined in the labor market of the agricultural goods sector and is equal to the marginal product of labor in the agricultural goods sector. Here we assume that households who reside in urban area and are not employed in manufactured goods sector do not have wage income, that is, w_u is equal to zero. Let L_c, L_r and L_u represent the number of each household type. Moreover, we normalize total population in the economy as one.

Since we assume that all households have common preference, they have the following same quasi-linear utility function as well as Ottaviano, Tabuchi, and Thisse[11].

$$U_l = \alpha \left(q_{0l} + \sum_{i=1}^{N} q_{il} \right) - \frac{\beta - \gamma}{2} \left([q_{0l}]^2 + \sum_{i=1}^{N} [q_{il}]^2 \right) - \frac{\gamma}{2} \left[q_{0l} + \sum_{i=1}^{N} q_{il} \right]^2 + z, \quad (l = c, r, u) \tag{1}$$

where q_{0l}, q_{il}, z, and N denote the consumption of goods produced by public firm, that produced by i-th private firm, and agricultural goods, the number of private firm in manufactured goods sector, respectively. As for parameters in (1), we assume that $\alpha > 0$ and $\beta > \gamma > 0$. α expresses the intensity of preference for differentiated manufactured goods. $\beta > \gamma$ means that consumers are biased toward a dispersed consumption varieties. Supposed that β is equal to γ, the substitutability between manufactured goods is perfect. [2]

Since all households has one unit of labor and $\bar{k}$ units of capital, they allocate their wage income and $r\bar{k}$ to consumption of manufactured goods and agricultural goods. Thus, the budget constraint of each households are given by

$$w_l + r\bar{k} = p_0 q_{0l} + \sum_{i=1}^{N} p_i q_{il} + z, \tag{2}$$

where p_0 and p_i are the price of differentiated goods and r is rent of capital. Maximizing utility function (1) subject to budget constraint (2), the first order

[2] As for detailed explanation of this utility function, see Ottaviano, Tabuchi, and Thisse[11].

condition of q_{0l} and q_{il} are as follows.

$$\alpha - (\beta - \gamma)q_{0l} - \gamma \left[q_{0l} + \sum_{i=1}^{N} q_{il} \right] - p_0 = 0 \tag{3}$$

and

$$\alpha - (\beta - \gamma)q_{il} - \gamma \left[q_{0l} + \sum_{i=1}^{N} q_{il} \right] - p_i = 0, \quad (i = 1, \cdots, N) \tag{4}$$

Solving (3) and (4), the optimal consumption of each differentiated goods is as follows.

$$q_{0l}^* = a - bp_{0l} + c \left[\sum_{j=0}^{N} [p_j - p_0] \right] \tag{5}$$

and

$$q_{jl}^* = a - bp_{jl} + c \left[\sum_{j=0}^{N} [p_j - p_0] \right], \tag{6}$$

where a, b, and c are $\alpha/[\beta + \gamma N]$, $1/[\beta + \gamma N]$, and $\gamma/(\beta - \gamma)[\beta + \gamma N]$, respectively. Since we assume that the symmetry of manufactured goods produced by private firms, let q_l represent the consumption of manufactured goods produced by private firm. Moreover, we define P as price index of manufactured goods market. So we can rewrite (5) and (6) as follows.

$$q_{0l}^* = a - [b + c(N + 1)]p_0 + cP \tag{7}$$

and

$$q_{il}^* = a - [b + c(N + 1)]p_i + cP \tag{8}$$

Here the price index P is defined as follows.

$$P \equiv \left[p_0 + \sum_{i=1}^{N} p_i \right] \tag{9}$$

Using (7) and (8), indirect utility function of household l is given as the function of consumption.

$$\begin{aligned} v_l = & \alpha \left(q_{0l}^* + \sum_{i=1}^{N} q_{il}^* \right) - \frac{\beta - \gamma}{2} \left([q_{0l}^*]^2 + \sum_{i=1}^{N} [q_{il}^*]^2 \right) - \frac{\gamma}{2} \left[q_{0l}^* + \sum_{i=1}^{N} q_{il}^* \right]^2 \\ & - p_0 q_{0l}^* - \sum_{i=1}^{N} p_i q_{il}^* + w_l + r\bar{k} \\ = & \frac{a^2(N+1)}{2b} - a \left(p_0 + \sum_{i=1}^{N} p_i \right) + \frac{b + c(N+1)}{2} \left([p_0]^2 + \sum_{i=1}^{N} [p_i]^2 \right) \\ & - \frac{c}{2} \left[p_0 + \sum_{i=1}^{N} p_i \right]^2 + w_l + r\bar{k} \end{aligned} \tag{10}$$

Though we assume that all households are mobile between regions, the wage in manufactured goods sector has downward rigidity attributable to the minimum wage system and so on and higher than the wage in agricultural goods sector. thus, each household compares the expected utility in urban area with the utility in rural area. Now we define λ as the unemployment rate in urban area, that is,

$$\lambda \equiv \frac{L_u}{L_c + L_u} \tag{11}$$

All households have no incentive to migrate between urban area and rural area when the expected utility in urban area is equal to the utility in rural area. Thus, migration equilibrium condition is given by the following equation.

$$(1-\lambda)v_c + \lambda v_u = v_r \quad \Leftrightarrow \quad (1-\lambda)\bar{w} = w_r \tag{12}$$

Since we assume that total population in the economy is normalized, the population constraint is as follows.

$$L_c + L_u + L_r = 1 \tag{13}$$

Combining (11) with (13), the population constraint in the economy is revised as follows.

$$L_c + (1-\lambda)L_r = 1 - \lambda \tag{14}$$

2.2 Production

2.2.1 Agricultural goods sector

We consider that two kinds of goods are produced in this economy. One of them is the agricultural goods and produced in rural area. On the other hand, the other is the manufactured goods and done in urban area. We assume that the market of agricultural goods is competitive. The agricultural goods sector has decreasing returns of scale with respect to labor. Particularly we specify the production function of agricultural goods sector as follows.

$$Z = (L_r)^{\sigma}, \quad \sigma \in (0,1) \tag{15}$$

The agricultural goods sector is competitive and the agricultural goods are homogeneous. Moreover, we assume that the wage has no downward rigidity in rural area. So the wage is equal to the marginal product in rural area.

$$w_r = \sigma(L_r)^{\sigma-1} \tag{16}$$

2.2.2 Manufacture goods sector

We consider that the manufacture goods sector in urban area is mixed oligopoly as well as Naito[9]. Though Naito[9] consider the mixed duopoly and *homogeneous* in the manufactured goods sector, we assume that the manufactured goods sector is the *differentiated* mixed oligopoly. In the mixed oligopoly market, the public firm competes with private firms in the common market. Here we consider that the manufactured goods market is the monopolistic competition as well as Ottaviano, Tabuchi, and Thisse [11]. Supposed that the number of private firm is so large, the behavior of firms in urban area does not affect the price index in the manufactured goods market regardless of public firm or private firms. First of all, we derive the total demand of each manufactured goods, which is denoted by $Q_j (j = 0, \cdots, N)$. Since the demand for each manufactured goods is independent of household's income due to quasi-linear function,

$$\begin{aligned} Q_0 &= (L_c + L_u + L_r)(q_{0c} + q_{0u} + q_{0r}) \\ &= a - [b + c(N+1)]p_0 + cP \end{aligned} \tag{17}$$

$$\begin{aligned} Q_i &= (L_c + L_u + L_r)(q_{ic} + q_{iu} + q_{ir}) \\ &= a - [b + c(N+1)]p_i + cP, \quad (i = 1, \cdots, N) \end{aligned} \tag{18}$$

As we have referred to the input factors of manufactured goods above explanation, we assume that each firm of manufactured goods has homogeneous production technology. the manufactured goods sector require the labor and capital as input factors for production. Supposed that Q_j units of production require L_{ic}/m units labor and one unit of capital, the cost functions of public firm and private firm are given by.

$$C_0(Q_0) = m\bar{w}Q_0 - r \tag{19}$$

and

$$C_i(Q_i) = m\bar{w}Q_i - r \quad (i = 1, \cdots, N), \tag{20}$$

where r denotes rent for one unit of capital. From (17) and (19), the profit function of public and private firm are given by the following π_0 and π_i.

$$\pi_0 = (a - [b + c(N+1)]p_0 + cP)(p_0 - m\bar{w}) - r \tag{21}$$

$$\pi_i = (a - [b + c(N+1)]p_i + cP)(p_i - m\bar{w}) - r \tag{22}$$

Though the private firms pursue their profit, the public firm determines his production to maximize the social welfare. Thus, we define the social welfare function to analyze the behavior of public firm. Let W represent the social welfare in this model as follows.

$$
\begin{aligned}
W = & \frac{a^2(N+1)}{2b} - a\left(p_0 + \sum_{i=1}^{N} p_i\right) + \frac{b + c(N+1)}{2}\left([p_0]^2 + \sum_{i=1}^{N}[p_i]^2\right) \\
& - \frac{c}{2}\left[p_0 + \sum_{i=1}^{N} p_i\right]^2 + \pi_0 + \sum_{i=1}^{N}\pi_i + \bar{w}(L_c^0 + \sum_{i=1}^{N} L_c^i) + \pi_r \\
& + w_r L_r + r(N+1) \qquad (23)
\end{aligned}
$$

Since we assume that the number of private firm is large as well as Ottaviano, Tabuchi, and Thisse [11], both public firm and private firms do not affect the price index of manufactured goods market. [3] Here we consider that the government owns the share of $(1-\theta)$ of public firm as well as Matsumura [7]. Thus, the purpose of public firm (firm 0) is to maximize the weighted average of social welfare and its profit, which is defined by V (θ). It is possible for us to take account of partial privatization as to optimal privatization level.

$$
\begin{aligned}
V(\theta) \equiv & \pi_0 + (1-\theta)W \\
& = (a - [b + c(N+1)]p_0 + cP)(p_0 - m\bar{w}) - r \\
& + (1-\theta)\left[\frac{a^2(N+1)}{2b} - a\left(p_0 + \sum_{i=1}^{N} p_i\right) + \frac{b + c(N+1)}{2}\right. \\
& \times \left([p_0]^2 + \sum_{i=1}^{N}[p_i]^2\right) - \frac{c}{2}\left[p_0 + \sum_{i=1}^{N} p_i\right]^2 + \sum_{i=1}^{N}\pi_i + \bar{w}(L_c^0 + \sum_{i=1}^{N} L_c^i) \\
& \left. + w_r L_r + r(N+1)\right], \qquad (24)
\end{aligned}
$$

where $\bar{L}_c$ denotes the number of households employed by public firm and private firms in urban area. Since each private firms determine the price to maximize their own profit function (22), the first order condition for profit maximization is given by

$$
\frac{\partial \pi_i}{\partial p_i} = a - 2[b + c(N+1)]p_i + cP + [b + c(N+1)]m\bar{w} = 0 \qquad (25)
$$

On the other hand, the public firm determines his price to maximize the weighted average of social welfare and its profit. The first order condition for the weighted average of social welfare and its profit is as follows. [4]

[3] Strictly speaking, the manufactured goods market is different from that in Ottaviano, Tabuchi, and Thisse [11]. This is because Ottaviano, Tabuchi, and Thisse [11] do not include public firm. In our model the public firm does not determine his product price for profit maximization. However, the behavior of public firm does not affect the price index of manufactured goods market. As for private firm, their behavior also affect the price index of it. Thus, the market in our model resembles a monopolistic competition market.

[4] Since the objective function of public firm also includes the profit of agricultural goods sector, the object function of public firm depends on the effect of public

$$\begin{aligned}\frac{\partial V(\theta)}{\partial p_0} =& \frac{\partial \pi_0}{\partial p_0} + (1-\theta)\left\{\frac{\partial CS}{\partial p_0} + \sum_{i=1}^{N}\frac{\partial \pi_i}{\partial p_0} + \bar{w}\frac{\partial L_c^0}{\partial p_0}\right\} \\ =& a - (1+\theta)[b + c(N+1)]p_0 + cP + \theta m\bar{w}[b + c(N+1)] = 0 \quad (26)\end{aligned}$$

From (25) and (26), the price of differentiated goods produced by either public firm or private firm is given as the function of Price index. Similar to Ottaviano, Tabuchi, and Thisse [11], the behavior of each firm does not affect this price index, that is, $\partial P/\partial p_j (j = 0, \cdots, N)$. Taking account of the symmetry of private firms, which is $p = p_1 = \cdots = p_N$. The equilibrium price of manufactured goods produced by private firm is derived as follows. Solving (25) and (26) with respect to p_0 and p_i, we derive p_0 and p_i as follows.

$$p_0 = \frac{a}{(1+\theta)\left[b + c(N+1)\right]} + \frac{c}{(1+\theta)\left[b + c(N+1)\right]}P + \left(\frac{\theta}{1+\theta}\right)m\bar{w} \quad (27)$$

$$p = \frac{a}{2\left[b + c(N+1)\right]} + \frac{c}{2\left[b + c(N+1)\right]}P + \frac{m\bar{w}}{2} \quad (28)$$

Taking account of the symmetry of private firms, we write price index as $P = p_0 + Np$. Substituting (27) and (28) into (9), the following equation is derived.

$$\begin{aligned}P =& p_0 + Np \\ =& \left(\frac{2 + (1+\theta)}{2(1+\theta)[b + c(N+1)]}\right)a + \left(\frac{2 + (1+\theta)}{2(1+\theta)[b + c(N+1)]}\right)cP \\ &+ \left(\frac{2\theta + (1+N)\theta}{2(1+\theta)}\right)m\bar{w} \quad (29)\end{aligned}$$

Solving (29) with respect to price index P, the equilibrium price index P^* is given by

$$P^* = \left(\frac{2 + (1+\theta)}{2\left(1+\theta\right)b + \left[2\theta + (1+\theta)N\right]c}\right)a + \left(\frac{2\left[2\theta + (1+\theta)\right]}{2\left(1+\theta\right)b + \left[2\theta + (1+\theta)N\right]c}\right)m\bar{w} \quad (30)$$

Substituting (30) into (27) and (28), we derive the equilibrium price of each firm, which is p_0^* or p^* as follows.

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firm's behavior on it via migration between urban area and rural area. However, we assume that the public firm does not take account of this effect to determine his production. Thus, the public firm deals with the profit of agricultural goods sector as given.

$$p_0^* = \left(\frac{2b + (N+3)c}{(b + (N+1)c)\left(2\left(1+\theta\right)b + \left[(1+\theta)\,N + 2\theta\right]c\right)} \right) a$$
$$+ \left[\frac{c}{(1+\theta)\left(b + c(N+1)\right)} \left(\frac{2\left(2\theta + (1+\theta)\right)}{2\left(1+\theta\right)b + \left(2\theta + (1+\theta)N\right)c} \right) + \frac{\theta}{1+\theta} \right]$$
$$\times m\bar{w} \tag{31}$$

$$p^* = \frac{(\theta+1)\,a}{2\left(1+\theta\right)b + c\left[N + (N+2)\,\theta\right]}$$
$$+ \left[\frac{c}{2\left[b + c(N+1)\right]} \left(\frac{4\theta + 2(1+\theta)N}{2\left(1+\theta\right)b + \left(2\theta + (1+\theta)\,N\right)c} \right) + \frac{1}{2} \right] m\bar{w} \tag{32}$$

Finally we refer to the number of private firms in the manufactured goods sector. Since every firm requires one unite of capital as a fixed input for production. Total number of manufactured goods sector is equal to $N+1$. On the other hand, every household in the economy has $\bar{k}$ units of capital as initial endowment and total number of households is one. Thus, the following equation must hold due to market clear condition in equilibrium.

$$N + 1 = \bar{k} \tag{33}$$

Solving (33) with respect to N, we derive the equilibrium number of private firms in the manufactured goods sector. [5] As for the number of private firms in the manufactured goods sector after this, we consider the situation where market clear condition of capital is held.

3 Migration between urban area and rural area

In previous we derive the behavior of households in the economy and production sectors of agricultural goods or manufactured goods sector. Though we partially show that the migration behavior of households between urban and rural area in subsection 2.1 Following Harris and Todaro[6], in our model households determine their residential area by comparing the expected utility in urban area with the utility in rural area.

Here we derive the number of employed households in urban area from the equilibrium. Let $\bar{Q}$ and Q represent the total number of production and each private firm production, respectively. Since $\bar{(Q)}$ is given by the sum of public firm production and private firms, $\bar{Q}$ is written as follows.

$$\bar{Q} = Q_0 + NQ = (N+1)a - bP \tag{34}$$

[5] Here we assume that $\bar{k}$ is larger than 2 and large enough.

Substituting (30) into (34), the equilibrium total production of manufactured goods is derived as follows.

$$\begin{aligned}\bar{Q} = &\left[(N+1) - b\left(\frac{3+\theta}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right)\right]a \\ &- b\left(\frac{2(1+3\theta)}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right)m\overline{w}\end{aligned} \tag{35}$$

Differentiating (35) with respect to m or $\bar{w}$, we have

$$\frac{\partial \bar{Q}}{\partial m} < 0, \quad \frac{\partial \bar{Q}}{\partial \bar{w}} < 0.$$

This results of comparative statics is unsurprising. The increase of productivity parameter m means that the production efficiency get worse. Thus, the increase of m leads to go up the production cost of manufactured goods sector and decrease the total production of them. As for $\bar{w}$, the increase of $\bar{w}$ decreases the total production because its increase also leads to rise the production cost. Moreover, total production $\bar{Q}$ depends on the privatization level of public firm, too. Since we assume that L_c units of labor produce L_c/m units of manufactured goods are produced, $m\bar{Q}$ units of labor are required to produce $\bar{Q}$ units of manufactured goods. So the employment in urban area depends on not only $\bar{w}$ but also the privatization level of public firm in the manufactured goods sector. Let $\bar{L}$ represent the labor demand of manufactured goods sector, that is,

$$\begin{aligned}\bar{L}_c = &\left[(N+1) - b\left(\frac{3+\theta}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right)\right]am \\ &- b\left(\frac{2(1+3\theta)}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right)m^2\overline{w}\end{aligned} \tag{36}$$

Differentiating $\bar{L}_c$ with respect to θ, we know the effect of privatization level of public firm on labor demand in the manufactured goods sector.

$$\frac{\partial \bar{L}_c}{\partial \theta} = \frac{[2+(3+N)c]\,a - 2\,[2+(N-1)c]}{(2\theta+2c\theta+Nc+Nc\theta+2)^2} \gtreqless 0 \tag{37}$$

The sign of $\partial \bar{L}_c/\partial\theta$ is not determined uniquely because it depends on the scale of $a(=\alpha/[\beta+\gamma N])$. Supposed that α expressing the intensity of preference for differentiated manufactured goods is large relatively, the value of a is also large relatively. When a is large relatively, the progress of privatization leads to increase the labor demand of manufactured goods sector because the numerator of $\partial \bar{L}_c/\partial\theta$ is positive. So we can derive the following Lemma.

Lemma 1

When the intensity of preference for differentiated manufactured goods is large relatively and a ***is larger than*** $[4+2(N-1)]/[2+(3+N)c]$, ***the progress of privatization leads to increase the labor demand of manufactured goods sector***.

moreover, we consider the migration equilibrium condition to analyze the effect of public firm's privatization on urban employment. Combining (12) with (16), we can derive the following equation.

$$L_r = (1-\lambda)^{\frac{1}{\sigma-1}} \left(\frac{\bar{w}}{\sigma}\right)^{\frac{1}{\sigma-1}} \tag{38}$$

Differentiating (38) with respect to $\bar{w}$, we know the following result of comparative statics, that is,

$$\frac{\partial L_r}{\partial \bar{w}} < 0.$$

This result of comparative statics is also obvious because the increase of minimum wage in urban area increases the expected utility in urban area. As the result, households have incentive to migrate from rural area to urban area. Finally substituting (36) and (38) into (14), the equilibrium unemployment in urban area is determined by the following equation.

$$\begin{aligned} &\left[(N+1) - b\left(\frac{3+\theta}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right)\right] am \\ &- b\left(\frac{2(1+3\theta)}{2(1+\theta)+(2\theta+(1+\theta)N)c}\right) m^2\overline{w} + (1-\lambda)^{\frac{\sigma}{\sigma-1}} \left(\frac{\bar{w}}{\sigma}\right)^{\frac{1}{\sigma-1}} \\ &= 1-\lambda \end{aligned} \tag{39}$$

Applying the implicit function theorem to (39) to analyze the effect of privatization of public firm on urban unemployment, $d\lambda/d\theta$ is derived as follows.

$$\begin{aligned} \frac{d\lambda}{d\theta} &= -\frac{2bm\left(\left[2+(N+3)c\right]a - 2\left[2+(N-1)c\right]m\bar{w}\right)}{\left[(2+(2+N)c)\theta+(2+cN)\right]^2\left[\left(\frac{\bar{w}}{\sigma}\right)^{\frac{1}{\sigma-1}}\left(\frac{\sigma}{1-\sigma}\right)(1-\lambda)^{\frac{1}{\sigma-1}}+1\right]} \\ &\gtreqless 0 \end{aligned} \tag{40}$$

Though the denominator in (40) is positive, the sign of numerator in ((40)) is not determined uniquely. The following inequalities must be established to hold the sign of numerator positive.

$$a > \frac{2c(N-1)+4}{c(N+3)+2} \tag{41}$$

From (41), the sign of numerator positive when the the intensity of preference for differentiated manufactured goods is large relatively. Thus, we know that the progress of privatization of public firm improve urban unemployment. Applying the implicit function theorem to (39) with respect to $\bar{w}$, we know that the increase of $\bar{w}$ increases the urban unemployment, that is,

$$\frac{\partial \lambda}{\partial \bar{w}} > 0.$$

Summarizing above comparative statics, we can derive the following proposition.

Proposition 1

When the intensity of preference for differentiated manufactured goods is large (small) relatively and a ***is larger(smaller) than*** $[4 + 2c\,(N-1)]m\bar{w}/[c\,(N+3)+2]$, ***the progress of public firm's privatization improves (makes a worse) urban unemployment.***

Next we consider the effect of public firm's privatization on social welfare in equilibrium. Since we define (23) as social welfare function, we substitute (31), (32), (30), and (39) into (23) to derive the equilibrium social welfare function. Let W^* represent the equilibrium social welfare function. Thus, the equilibrium social welfare function is as follows.

$$W^* = \frac{a^2(N+1)}{2b} - \frac{b + c(N+1)}{2}\left([p_0^*]^2 + N[p^*]^2\right) + \frac{c}{2}\left[P^*\right]^2 + (L_r)^\sigma \qquad (42)$$

Differentiating (42) with respect to *theta*, the effect of public firm's privatization on social welfare in equilibrium is as follows.

$$\frac{\partial W^*}{\partial \theta} = -[b + c(N+1)]\left(p_0^* \frac{\partial p_0^*}{\partial \theta} + N p^* \frac{\partial p^*}{\partial \theta}\right) + cP^* \frac{\partial P^*}{\partial \theta} + \frac{\partial}{\partial \theta}\left((L_r)^\sigma\right) \qquad (43)$$

Here we define the first term, the second term, and third term as price effect, price index effect, and employment effect, respectively. Generally, the sign of (43) is not determined uniquely. Supposed that a is large enough, $\partial p_0^*/\partial\theta$, $\partial p^*/\partial\theta$, and $\partial P^*/\partial\theta$ are negative. Moreover, the sign of (40) is positive as long as a is large enough and (41) is held. In this case the price effect is positive though price index effect is negative.

As for employment effect, the sign of this term is positive. If the price effect and employment effect exceed the price index effect under the case, in which a is large enough, the progress of public firm's privatization leads to improve

the social welfare. [6] Thus, we derive the following proposition.

Proposition 2

When a is large enough and the price effect and the employment effect exceed the price index effect, the progress of privatization of public firm improves the social welfare.

4 Concluding remarks

Most of traditional dualistic economy models assume that both urban sector (manufactured goods sector) and rural sector (agricultural goods sector) are competitive. However, we consider the economy, in which the manufactured goods sector is a differentiated mixed oligopoly. Though Naito[9] also consider the model including the mixed duopoly and dualistic economy models, he does not refer to the product differentiation. On the other hand, though Fujiwara[4] introduces the product differentiation into the mixed oligopoly model, his model does not consider the dualistic economy. Moreover, though Fujiwara[4] adopts the quasi-linear utility function used in Ottaviano, Tabuchi, and Thisse[11] to describe the product differentiation, that model does not take account of monopolistic competition market of manufactured goods in Ottaviano, Tabuchi, and Thisse[11]. Thus, our model supplements a blank of these previous studies.

We construct the simple model by synthesizing Fujiwara[4] and Naito[9] and analyze how the privatization of public firm affects urban unemployment or social welfare. Generally, the effect of privatization of public firm on urban unemployment is not determined uniquely. In our model the intensity of preference for differentiated manufactured goods play an important role. Supposed that the intensity of preference for differentiated manufactured goods is large relatively, the parameter a is large relatively. From (37) the progress of privatization of public firm increases labor demand in the manufactured goods sector. If the inequality holds in (41), the progress of it leads to worse urban unemployment. Namely, the unemployment rate in urban area rises though labor demand increased by privatization. Moreover, we analyze the effect of privatization on social welfare. The effect depends on the each parameter and is not determined uniquely. Then a price effect and an employment effect exceed a price index effect under the enough large a, the privatization of the public firm improve the social welfare.

[6] See Appendix A and Appendix B about the price effect, price index effect, and employment effect in detail.

Finally we do not derive the effect of product differentiation parameter γ on urban employment or social welfare analytically because of solution complexness. It is necessary to make a use of numerical analysis with simulation. We would like to analyze those points as a future subject.

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A Comparative statics of (30), (31), and (32) with respect to θ

Differentiating (30) with respect to θ, we derive the followin derivative, that is,

$$\frac{\partial P^*}{\partial \theta} = -\frac{2\left[(2b+(N+3)c)\,a - 2\,(2b+(N-1)c)\,mw\right]}{(2b+2b\theta+2c\theta+Nc+Nc\theta)^2} \tag{A.1}$$

Differentiating (31) with respect to θ, we derive the followin derivative, that is,

$$\begin{aligned}\frac{\partial p_0^*}{\partial \theta} =& \frac{-a\,(2b+(2+N)c)\,(2b+(3+N)c)}{(b+(N+1)c)\,(2b+2b\theta+2c\theta+Nc+Nc\theta)^2} \\ &+ \left[\frac{-2c\left[3\,(2b+(2+N)c)\,\theta^2 + 2\,(2b+(2+N)c)\,\theta - (2b+(N-2)c)\right]}{(\theta+1)^2\,(b+c+Nc)\,(2b+2b\theta+2c\theta+Nc+Nc\theta)^2}\right. \\ &\left. + \frac{1}{(\theta+1)^2}\right] m\bar{w}. \end{aligned} \tag{A.2}$$

As for (32), we differentiate (32) with respect to θ. The derivative is as follows.

$$\frac{\partial p^*}{\partial \theta} = \frac{-2c}{b+c+Nc}\frac{(b+(N+1)c)\,a - 2bm\bar{w}}{(2b+2b\theta+2c\theta+Nc+Nc\theta)^2} \tag{A.3}$$

Moreover, we estimate (A.1), (A.2), and (A.3) at $\theta = 0$ or $\theta = 1$, respectively.

$$\frac{\partial P^*}{\partial \theta}|_{\theta=0} = -\frac{2\left[(2b+(N+3)c)\,a - 2\,(2b+(N-1)c)\,mw\right]}{(2b+Nc)^2}, \tag{A.4}$$

$$\frac{\partial P^*}{\partial \theta}|_{\theta=1} = -\frac{2\left[(2b+(N+3)c)\,a - 2\,(2b+(N-1)c)\,mw\right]}{4\,(2b+(N+1)c)^2}, \tag{A.5}$$

$$\begin{aligned}\frac{\partial p_0^*}{\partial \theta}|_{\theta=0} =& \frac{-a\,(2b+(2+N)c)\,(2b+(3+N)c)}{(b+(N+1)c)\,(2b+Nc)^2} \\ &+ \left[\frac{2c\left[(2b+(N-2)c)\right]}{(b+c+Nc)\,(2b+Nc)^2} + 1\right] m\bar{w}, \end{aligned} \tag{A.6}$$

$$\begin{aligned}\frac{\partial p_0^*}{\partial \theta}|_{\theta=1} =& \frac{-a\,(2b+(2+N)c)\,(2b+(3+N)c)}{4\,(b+(N+1)c)\,(2b+(N+1)c)^2} \\ &+ \left[\frac{-2c\,(2b+(3+N)c)}{(\theta+1)^2\,(b+(N+1)c)\,(2b+(N+1)c)^2} + \frac{1}{4}\right] m\bar{w}, \end{aligned} \tag{A.7}$$

$$\frac{\partial p^*}{\partial \theta}|_{\theta=0} = \frac{-2c}{b+c+Nc}\frac{[(b+(N+1)c)\,a - 2bm\bar{w}]}{(2b+Nc)^2}, \tag{A.8}$$

and

$$\frac{\partial p^*}{\partial \theta}|_{\theta=1} = \frac{-2c}{b+c+Nc}\frac{[(b+(N+1)c)\,a - 2bm\bar{w}]}{4\,(2b+(N+1)c)^2}. \tag{A.9}$$

B Comparative statics of market effect with respect to θ

Substituting (38) into (15), $(L_r)^\sigma$ is given by

$$(L_r)^\sigma = (1-\lambda)^{\frac{\sigma}{\sigma-1}}\left(\frac{\bar{w}}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}. \tag{B.1}$$

Differentiating (B.1) with respect to θ,

$$\frac{\partial}{\partial \theta}(L_r)^\sigma = \frac{\sigma\,(1-\lambda)^{\frac{\sigma}{\sigma-1}}\left(\frac{w}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}}{(1-\sigma)\,(1-\lambda)}\frac{\partial \lambda}{\partial \theta} \tag{B.2}$$

$$\frac{\sigma\,(1-\lambda)^{\frac{\sigma}{\sigma-1}}\left(\frac{w}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}}{(1-\sigma)\,(1-\lambda)} > 0 \tag{B.3}$$

Here, supposed that a is large enough, $\frac{\partial \lambda}{\partial \theta}$ is positive. If a is large enough, the employment effect is positive.