Bank Runs and Interbank Markets: A Heuristic Example

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Abstract

This paper offers an example of the game in which banks make lending and borrowing decisions while depositors choose when to withdraw, to better understand how the interbank market rescues a bank at risk of bank runs. Our example is based on Postlewaite & Vives (1987) and adds banks and the interbank market. We show that there is a Bayesian perfect equilibrium in which the interbank market collapses and a bank run occurs. There is also an equilibrium where banks refuse to lend and a bank run happens when a situation of the bank run (strategic withdrawal) is associated with a situation of default of the rescue loan. Therefore, the interbank market alone does not necessarily save a bank at risk of bank runs.

Key Words: Bank runs, Interbank Markets, Coordination Failure, Bayesian Perfect Equilibrium, Mismatch of Maturity between Lending and Borrowing

Ⅰ. Introduction

A bank run has been a central issue of financial crisis. It is not a long ago that economists began to develop theoretical models of bank runs. Since Diamond & Dybvig (1983), a bank run phenomenon has been understood as "sun-spot equilibrium" induced by coordination failure among depositors. (See Freixas, X. & J-C. Rochet (2008, Ch.7) and Sakai & Maeda (2003), for example.) At the same time, various policies that can prevent a bank run from occurring have been discussed from reserve and equity capital ratio requirements to suspension of convertibility.

A role of a central bank as the lender of last resort (a rescue loan to a bank at risk of bank runs) also has been debated. Goodfriend & King (1988), comparing private bank loans with a central bank's discount window policy, conclude that there is no compelling reason why the central bank is superior to private banks in terms of monitoring and assessing a borrowing bank, by analogy with a comparison between private and public firms in goods markets. To my knowledge, there has been no theoretical model where private banks in interbank markets can successfully rescue an illiquid but solvent bank. On the other hand, some economists, suspecting a chain reaction of bank runs through financial markets, developed their models to support the central bank's role of the lender of last resort. Most of them (for example, Allen, F. & D. Gale (2000), Freixas, X., Parigi, B. M. & J-C. Rochet (2000), Freixas, X., Martin, A. & D. Skeie (2011)) are based on Diamond & Dybvig (1983), while Rochet & Vives (2004) is based on Postlewaite & Vives (1987), with global game structure under incomplete information (Morris, S. & H. S. Shin (1998)).

In these models, banks' lending and borrowing actions after risk of a bank run is perceived are not analyzed because all banks' actions are treated in ex ante optimal deposit contract (as a maximization problem of depositor's expected utility subject to a bank's zero profit condition). This structure makes it difficult to explicitly analyze how banks behave in the interbank market when a bank faces a risk of a bank run (strategic withdrawals). As a result, these models do not explicitly explain why the interbank market would not work to rescue a bank at risk of bank runs (in other words, what kind of market failure happens in the interbank market), but rather focus on working of policy instruments such as reserve and equity capital ratio requirements and the central bank's lending to the bank (discount window policy). Furthermore, these models are complicated and not explicitly stated as a game form. This makes it hard for the reader to understand these models.

In this paper, I attempt to offer a heuristic example to better understand how private banks would (fail to) provide liquidity to a troubled bank, at the same time taking into account how its depositors withdraw strategically.

Participants in the interbank market are limited to a certain group of banks and other financial institutions. It is easy and costless for them to participate in and retreat from the market since what is traded is a short-term loan (or its rolling over). Thanks to new technology of information delivery and processing, some of them are likely to have almost the same information so that their actions are coordinated within their group, as in Morris, S. & H. S. Shin (1998). This makes it appropriate to analyze the interbank market as a game theoretical situation rather than a competitive market, even if there appears many market participants.

In our example, we add banks as players to the game of strategic withdrawal of Postlewaite & Vives (1987). There banks receive signals and choose their loans demanded and offered, then agents (depositors) at a troubled bank receive signals of their own types and choose when to withdraw. The novelty of our example is our formulation of the troubled bank's objective and the interbank market. Following a separation of ownership from management, we assume that the bank tries to avoid its bank run (whenever such a risk exists), i,e, its manager tries to minimize his expected penalty of a bank run. By doing so, we can avoid the deadlock in the existing literature of being unable to explicitly treat banks' behavior in the interbank market together with strategic withdrawals. To avoid a complicated matching mechanism in the interbank market, we simply assume that the interbank market determines the loan volume at the short-side when excess demand (supply) happens at the lowest (highest) interest rate and assume that the interest rate adjusts quickly in response to excess demand. Though the interest rate may be indeterminate when demand and supply are balanced, it does not affect our results since we can assign either the highest or lowest rate in this case.

Our example shows two types of equilibrium. One is an equilibrium where the interbank market collapses with no trade and a bank run occurs. The other is an equilibrium where a bank run happens when a rescue loan is expected to default (due to mismatch of maturity between lending and borrowing). What happens in the first type of equilibrium is a typical case of coordination failure among banks. Both lenders and borrowers have pessimistic expectations and shrink to avoid their transactions. The second type of equilibrium suggests that the ultimate reason for default of the rescue loan (and thus the bank run) results from mismatch of maturity between lending and borrowing, not from insolvency of a bank (or a financial institution) or sudden devaluation of the deposits or liquid assets held at banks due to exogenous shocks. This implies that we cannot avoid a bank run under the modern partial reserve banking system.

This paper is organized as follows. In the next section we formulate an example of the game among depositors and banks. In section 3, equilibrium of the game is analyzed and its economic logic is discussed. We conclude by making some remarks on robustness and possible extensions of our example.

Ⅱ. The Example

Our example is a modified version of the example in Postlewaite & Vives (1987). We add n+1 banks (managers of these banks) as players.

(1) Technology and Environments

There are four periods, 0, 1, 2, and 3. The production process exhibits constant returns to scale. For each unit planted at period 0, there will be α units available after one period, β units available after two periods, and γ units available after three periods. For each unit left in period 1 of some amount planted in period 0 there will be β/α units in period 2 and γ/α units in period 3. For each unit left in period 2 of some amount planted in period 0 there will be γ/β units one period later. We assume that

$$\alpha < \beta < \gamma,\ 1/2 < \alpha < 1, \text{ and } \gamma > 1. \quad (1)$$

If the production process is interrupted in period 1, one gets α, which is less than the initial investment 1. If the production process is not interrupted, one gets $\gamma > 1$. If it is interrupted in period 2, one gets β, which is larger than α but less than γ.

In our example, we explicitly treat strategic behavior of withdrawal by agents at one of the banks, denoted by bank 0, which is in a situation of a bank run.

At bank 0, there are two agents, each of whom has one unit of endowment and will live for two or three periods. At the beginning of period 1, agent i ($i = a$ or b) receives a signal $s^i \in S^i$, $S^i = \{s^i_2, s^i_3\}$, which indicates his or her life span (type). We call an agent who lives for two periods *short-lived*, an agent for three periods *long-lived*. There is a joint distribution P on $S^a \times S^b$. Each agent has his utility function: $U(\sum_{i=1}^{\tau} x_i)$, where τ is the number of periods the agent will live and x_i the consumption in period i.

(2) Banking Contract

Each agent deposits an amount in bank 0. During periods 1 and 2, an agent can withdraw his deposit with no interest (penalty for early withdrawal). If the agent withdraws his deposit in period 3, he receives a share of what is left in the bank proportional to his deposit. If, during any period, demand for withdrawal exceed assets, all assets will be distributed proportional to withdrawal demands.

(3) Banks and Interbank Market

There are $n+1$ banks in the interbank market ($n \geq 2$). They are only participants in it. Without loss of generality, we assume that bank 0 is in a situation of bank runs without borrowing in the interbank market. Bank i receives signal s^i at the beginning of period 1 ($i = 0, 1, 2, \cdots, n$). The signals that banks receive indicate what pair of agent type at bank 0 happens. We assume that the signals and the type pairs are perfectly correlated.[1] (This means that all participants in the interbank market know a situation of the bank at risk of bank runs.) $s^i \in S \equiv \{(s^a_2, s^b_2), (s^a_2, s^b_3), (s^a_3, s^b_2), (s^a_3, s^b_3)\}$ for $i = 0, 1, 2, \cdots, n$. After receiving signals, banks decide how much to borrow or lend in period 1.

Bank 0 has initially no liquid asset. Its strategy is how much per agent to borrow from the interbank market for each signal, denoted by $d(s^0) \in D \equiv [0, \infty)$. Other banks are assumed to have liquid assets and be immune to any bank run even if they lend whatever amounts of their

1) In other words, banks know types of both agents at bank 0 at the beginning of period 1.

liquid assets to bank 0. (We strategically assume away a chain reaction of bank runs through the interbank markets and instead focus on whether other banks (or financial institutions) participate in transactions in the interbank market.) Bank i ($i = 1, 2, \cdots, n$) decides how much to lend to bank 0 for each signal, denoted by $l_i(s^i) \in L_i \equiv [0, m_i]$ where m_i is the amount of liquid asset bank i holds at the beginning of period 1. All banks make their decisions on borrowing and lending simultaneously.

The loan in the interbank market is borrowed in period 1 and repaid with its interest $R-1$ in period 3. The loan in question is of a rescue loan so that there is no opportunity for bank 0 to access to any financial market in period 2 and that the lenders will have to wait until period 3 when bank 0 will get a higher yield of its (long-run) production. The interbank market is short-sided constrained (though the interest rate responds to excess demand), "anonymous" (except that bank 0 is at risk of bank runs and the only borrower) in the sense that the aggregate supply and demand, not identity of participants, affect the interest rate) and "centralized" in the sense that all demands and supplies are matched at one place. (We assume away a favorite relationship between bank 0 and some other banks.) However, the number of banks potentially participating in the interbank market is limited so that the interaction among the banks matters. Note that since we assume constant-returns-to-scale technology, demand for loans may be infinity, which is eventually limited by a fixed amount of liquid assets possessed by banks at the maximum price. When excess supply happens at the minimum price, the demand side determines the amount of loan traded (and the supply side is rationed in proportion to the initial offers). The interest rate is adjusted so as to equate demand with supply ($2d = \sum_{i=1}^{n} l_i$ for $1 < R < \gamma/\alpha$, $2d \leq \sum_{i=1}^{n} l_i$ for $R=1$ and $2d \geq \sum_{i=1}^{n} l_i$ for $R = \gamma/\alpha$) in the interbank market. We assume that $1 \leq R \leq \max\{(\beta/\alpha), (\gamma/\alpha), (\gamma/\beta)\} = (\gamma/\alpha)$ (maximum willingness to pay for the loan), though we will not explicitly analyze the equilibrium interest rate in some cases.

The banking contract, implicitly made in a competitive deposit market, is supposed to leave zero profit to a manager of a bank in the literature. This makes it difficult to formulate a bank's objective function in a context of strategic withdrawal. We assume that a manager of bank 0 tries to avoid a bank run, i.e., to maximize expected payoff of $(-1) \times P_0 + (0) \times (1 - P_0)$, where P_0 is the probability of bank 0's bank run, which depends on the pair of strategy of agents at bank 0 specified below. As for other banks, however, we assume that bank i's payoff (or its manager's payoff) is its expected yield of lending in the interbank market. For $i = 1, 2, \cdots, n$, $(R-1)\, l_i$ if both agents repay their loans in period 3 (both live until period 3) or at least one of agents (who is long-lived) repays the loans of the bank, $(1/2)\{(R-1)\, l_i + (-l_i)\}$ if one of the agents fails to repay (he lives until period 2), and $-l_i$ if both fail to repay the loans (they are short-lived). This asymmetric treatment between bank 0 and other banks may be justified by interpreting a manager's preference as a lexicographic one: he first tries to avoid a bank run whenever such a risk exists, but otherwise tries to maximize the bank's expected yield of loan.

(4) Agents at bank 0

An agent at bank 0 decides when to withdraw his deposits after receiving his signal (type) and observing how much banks have borrowed and lent. A strategy for an agent is a function σ^i, which indicates when agent i will withdraw his money from bank 0 for each possible signal he receives and each possible combination of lending and borrowing amounts of banks. $\sigma^i : S^i \times \mathrm{D} \times \prod_{i=1}^{n} L_i \rightarrow A$, where $A = \{w_1, w_2, w_3\}$ and w_i represents withdrawal in period i. Agent i's belief of agent j's type is denoted by $\mu^i(s_k^j \mid s_{k'}^i, d, l)$, where $l = (l_1, l_2, \cdots, l_n)$.

Table 1 shows payoffs to agents at bank 0 when there is neither borrowing nor lending ($d(s^0) = 0$, $l_i(s^i) = 0$ for $i = 1, 2, \cdots, n$).[2] Note that a strategy of withdrawal in period 3 is not available to s_2^i type (short-lived) player.

We make the following assumptions on parameters in Table 1 to assure that bank 0 is in a situation of bank runs without borrowing and lending.

Table 1: Payoff Matrix of Agents at Bank 0 without borrowing and lending

Agent a/Agent b	w_1	w_2	w_3
w_1	α, α	$1, (2\alpha-1)(\beta/\alpha)$	$1, (2\alpha-1)(\gamma/\alpha)$
w_2	$(2\alpha-1)(\beta/\alpha), 1$	β, β	$1, (2\beta-1)(\gamma/\beta)$
w_3	$(2\alpha-1)(\gamma/\alpha), 1$	$(2\beta-1)(\gamma/\beta), 1$	γ, γ

$$\beta < 1,\ (2\alpha-1)(\beta/\alpha) < \alpha < (2\alpha-1)(\gamma/\alpha),\ 1 < (2\beta-1)(\gamma/\beta). \qquad (2)$$

Under the above conditions, both short-lived agents have a dominant strategy of withdrawal in period 1, while both long-lived agents have a dominant strategy of withdrawal in period 3 in the payoff matrix of Table 1. This implies that a bank run occurs (where both agents withdraw their deposits in period 1) whenever both agents are short-lived, as demonstrated in Postlewaite & Vives (1987).

When bank 0 borrows d (per depositor) from the interbank market, payoffs to agents at bank 0 must be classified into the four possible cases of types. Though claims and debts are in the agent's hand and carried over to period 3 when bank 0 is resolved, short-lived agents can never pay their debts in period 3 because they will no longer live in period 3. The debt the short-lived agent carries over to period 3 will be default. This makes payoffs different, depending on what pair of types occurs.

2) This table is the same as Table 1 in Postlewaite & Vives (1987).

Let $A(x, y, d) \equiv (2x-1+2d)(y/x)$, $B(x, y, d) \equiv (2x-1+2d)(y/x)-2Rd$ and $C(x, d) \equiv x+(1-R)d$. We assume that $d < 1/2$. Otherwise a bank will be able to meet unilateral withdrawal without liquidating part of its long run investment. Each cell of Table 2 is divided into the four possible states of type, in each of which payoffs to agents are shown. The payoffs in case of both players being short-lived is in the northwest, those for short-lived agent a and long-lived agent b in the northeast, those for long-lived agent a and short-lived agent b in the southwest, and those in case of both being long-lived in the southeast. The dash (-) indicates that there is no possible strategy pair since at least one agent is short-lived in the cell. When d=0, Table 2 coincides with Table 1.

Table 2: Payoff Matrix of Agents at Bank 0 with borrowing and lending

a/b	w_1		w_2		w_3	
w_1	$d+\alpha, d+\alpha$	$d+\alpha, C(\alpha, d)$	$1, A(\alpha, \beta, d)$	$1, B(\alpha, \beta, d)$	-	$1, B(\alpha, \gamma, d)$
	$C(\alpha, d), d+\alpha$	$C(\alpha, d), C(\alpha, d)$	$1, A(\alpha, \beta, d)$	$1, B(\alpha, \beta, d)$	-	$1, B(\alpha, \gamma, d)$
w_2	$A(\alpha, \beta, d), 1$	$A(\alpha, \beta, d), 1$	$d+\beta, d+\beta$	$d+\beta, C(\beta, d)$	-	$1, B(\beta, \gamma, d)$
	$B(\alpha, \beta, d), 1$	$B(\alpha, \beta, d), 1$	$C(\beta, d), d+\beta$	$C(\beta, d), C(\beta, d)$	-	$1, B(\beta, \gamma, d)$
w_3	-	-	-	-	-	-
	$B(\alpha, \gamma, d), 1$	$B(\alpha, \gamma, d), 1$	$B(\beta, \gamma, d), 1$	$B(\beta, \gamma, d), 1$	-	$C(\gamma, d), C(\gamma, d)$

(5) A Summary of the Game

The players are agent a and agent b at bank 0, managers of the banks numbered from zero to n. The timing of the game is as follows. Each player receives his signal (on his type for agents and on a pair of type for banks) at the beginning of period 1. Then banks borrows or lends in the interbank market. After observing these transactions, agents at bank 0 decides when to withdraw.

Agent i's strategy is when to withdraw, given his type and transactions in the interbank market: $\sigma^i : S^i \times B \times \prod_{i=1}^{n} L_i \rightarrow A$, where $A = \{w_1, w_2, w_3\}$ and w_i represents withdrawal in period i. Agent i's belief of agent j's type, $\mu_i(s^j_k \mid s^i_{k'}, d, l)$ is updated if possible, where $k, k' = 2, 3$ and $l = (l_1, l_2, \cdots, l_n)$. A manager of bank 0 has a strategy of how much to borrow, given his received signal on a pair of type: $d(s^0)$: $S \rightarrow D \equiv [0,\infty)$. A manager of bank $i\,(i = 1, 2, \cdots, n)$ has a strategy of how much to lend, given her received signal: $l(s^i)$: $S \rightarrow L_i \equiv [0, m_i]$ where m_i is the amount of liquid asset bank i holds at the beginning of period 1. Banks' beliefs of a pair of type are omitted because signals are perfectly correlated with actual pairs of type.

Payoff to agent i ($i = a, b$): Let $\pi_i(\sigma^i(s^i, d, l), \sigma^j(s^j, d, l)$: $s^i_k, s^j_{k'})$ be agent i's payoff function when he is type k and the other agent is type k' where $k, k' = 2, 3$, which appears in Table 2. For ($s^i_{k'}$, d, l), his expected payoff is

$$\sum_{k'=2,3} \pi_i(\sigma^i(s^i, d, l), \sigma^j(s^j, d, l): s^i_{k'}, s^j_{k}) \, \mu_i(s^j_{k'} \mid s^i_{k'}, d, l).$$

Payoff to bank 0: Let $P_0(\sigma^i(s^i, d, l), \sigma^j(s^j, d, l))$ be the probability that a bank run occurs at bank 0. For (s^i, s^j), bank 0's (its manager's) expected payoff is

$$(-1)\times P_0(\sigma^i(s^i, d, l), \sigma^j(s^j, d, l)) + (0)\times(1 - P_0(\sigma^i(s^i, d, l), \sigma^j(s^j, d, l))).$$

Payoff to bank i ($i = 1, 2, \cdots, n$): For (s^a, s^b), bank i's expected payoff is [3)]

$$\begin{array}{ll} -l_i & \text{if } (s^a, s^b) = (s_2^a, s_2^b) \text{ and } \sigma^i(s^i, d, l) = w_1 \text{ or } w_2 \text{ for } i = a, b, \\ (1/2)\{(R-1)\,l_i + (-l_i)\} & \text{if } (s^a, s^b) = (s_2^a, s_3^b) \text{ and } \sigma^b(s^b, d, l) = w_2, \\ (1/2)\{(R-1)\,l_i + (-l_i)\} & \text{if } (s^a, s^b) = (s_3^a, s_2^b) \text{ and } \sigma^a(s^a, d, l) = w_2, \\ (R-1)\,l_i & \text{otherwise.} \end{array}$$

The equilibrium concept we employ is Bayesian Perfect Equilibrium.

Ⅲ. Equilibrium and Its Economic Logic

In this section, we show that there is an equilibrium in which a bank run happens with positive probability even if all the other banks in the interbank market have no risk of their own bank runs. In other words, our example demonstrates that the interbank market alone does not necessarily work to avoid a bank run.

Let $M \equiv (1/2)\sum_{i=1}^{n} m_i$, $\overline{m} \equiv (1/2)\max\{m_1, m_2, \cdots, m_n\}$ and $t \equiv \min\{d, (1/2)\sum_{i=1}^{n} l_i\}$, as the maximum amount of loan (per depositor) made by all banks, the maximum amount of loan (per depositor) made by an individual bank. and the realized amount of loan (per depositor) determined by the short side between demand and supply, respectively. We define

$t_1 \equiv 1-\beta$, $t_2 \equiv \{\alpha^2 - (2\alpha-1)\beta\}/(2\beta-\alpha)$,

$t_3 \equiv \min\{(\gamma-1)/(R-1), (\beta-(2\beta-1)\gamma)/(\gamma-R\beta)\}$ for $R \neq 1$ if $R > \gamma/\beta$.

Otherwise $t_3 \equiv (\gamma-1)/(R-1)$.

Proposition 1

Suppose that a joint probability of (s_2^a, s_2^b) is positive. In addition to Assumptions (1) and (2), we make the following assumptions.

3) For ease of understanding, we separately describe payoffs depending on the signals received since we assume that signals and the actual pair of type are perfectly correlated.

(a) For all $t \leq \overline{m}$ and for all R $(1 \leq R \leq \gamma/\alpha)$,

(i) $t + \alpha > A(\alpha, \beta, t)$, (ii) $1 > t + \beta$, (iii) $C(\alpha, t) < B(\alpha, \gamma, t)$,

(iv) $\max\{C(\beta, t), 1\} < B(\beta, \gamma, t)$, (v) $1 < C(\gamma, t)$.

(b) (vi) $t_1 < t_2 < t_3$, (vii) $t_2 < M < t_3$.

Then there is an equilibrium in which a bank run happens with the joint probability of (s_2^a, s_2^b), and equilibrium strategies are as follows.

For $i = a, b$,

$\sigma^i(s_2^i, d, l) = w_1$, with any belief, $\sigma^i(s_3^i, d, l) = w_3$, with any belief, for (d, l) such that $t < t_1$,

$\sigma^i(s_2^i, d, l) = w_1$, with $\mu_i(s_{k'}^j \mid s_k^i, d, l) = 0$ ($k' \neq k$ and $j \neq i$) for (d, l) such that $t_1 \leq t \leq t_2$,

$\sigma^i(s_3^i, d, l) = w_3$, with any belief, for (d, l) such that $t_1 \leq t \leq t_2$,

$\sigma^i(s_2^i, d, l) = w_2$, with any belief, $\sigma^i(s_3^i, d, l) = w_3$, with any belief, for (d, l) such that $t_2 < t \leq M$.

$$d(s^0) = 0 \quad \text{for all } s^0, \quad l_i(s^i) = 0 \quad \text{for all } s^i, (i = 1, 2, \cdots, n).$$

Proof: Note that the assumptions in Proposition 1 are satisfied for some combinations of parameters if we take γ large enough and β close to one, and m_i $(i = 1, 2, \cdots, n)$ appropriately.

For agent i $(i = a, b)$, they face the payoff matrix of Table 2 where d is replaced with t. When $t < t_1$, the above conditions (i) to (v) hold. So a short-lived agent has his dominant strategy of w_1, while a long-lived agent has her dominant strategy of w_3. In this case, we can allow any belief of type. When $t_1 < t < t_2$, inequality of condition (ii) is reversed. Since any arbitrarily specified belief of type is allowed in Bayesian perfect equilibrium, we specify the belief that the other agent is of the same type. Under this belief, a short-lived agent thinks that he plays the two-by-two northwest payoff matrix, in which there are two equilibria (w_1, w_1) and (w_2, w_2). We choose the former as the equilibrium path. A long-lived agent has his dominant strategy of w_3 (with any belief) when $t_1 < t < t_2$. (We employ weak dominance so that we can include the cases of equality in the second range of t without changing weakly dominant strategies.) When $t_2 < t \leq M < t_3$, inequalities of conditions (ii) and (i) are reversed. A short-lived agent has his dominant strategy of w_2, while a long-lived agent has her dominant strategy of w_3.

Given $l_i(s^i) = 0$ for all s^i $(i = 1, 2, \cdots, n)$, we have $t = 0$ whatever amount of loan bank 0 demands. Thus a bank run happens only when the type pair is (s_2^a, s_2^b). Bank 0 cannot change the probability of bank runs so that $d(s^0) = 0$ for all s^0 is its best response. Given $d(s^0) = 0$ for all s^0, any other bank cannot change $t = 0$ whatever amount of loan it offers. Thus $l_i(s^i) = 0$ for all s^i $(i = 1, 2, \cdots, n)$ are their best responses. Q.E.D.

What happens in equilibrium of Proposition 1 is a typical case of coordination failure among the participants in the interbank market, even if bank 0 faces risk of a bank run. Bank 0 expects other banks to lend no loan so that it demands no loan. Other banks expect bank 0 to demand no loan so that they offer no loan. These expectations are realized in equilibrium. As a result, there occurs no transaction in the interbank market, which eventually collapses. This happens not only when a bank run occurs (both agents are short-lived) but also in all the other cases. Whether this kind of coordination failure takes place is independent of whether bank 0 has risk of bank runs. Rather this kind of coordination failure always happens whenever the number of market participants is limited and their expectations are likely to converge (as perfectly correlated signals in our example).

In equilibrium of Proposition 1, the interbank market collapses so that the interest rate is not determined.

The reader might wonder that the equilibrium in Proposition 1 may be pathological in the sense that bank 0 demands no loan in the interbank market. The next proposition shows another equilibrium in which bank 0 demands a positive amount of loan.

Proposition 2

Suppose that all the assumptions in Proposition 1 are satisfied. Suppose further that

(viii) $t_2 < M - \overline{m} < t_3$.

Then there is an equilibrium in which a bank run happens with the joint probability of (s_2^a, s_2^b), and equilibrium strategies are the same as those in Proposition 1 for agents at bank 0 and those of banks are

$d(s^0) = d > M$ for all s^0.

For $i = 1, 2, \cdots, n$, $l_i(s^i) = 0$ if $s^i = (s_2^a, s_2^b)$, $l_i(s^i) = m_i$ otherwise.

Proof: Note that the additional condition (viii) is satisfied if we take m_i and n appropriately.

The proof of equilibrium strategies for agents at bank 0 is the same as that in Proposition 1, because what is relevant to agents' strategies is the realized amount of loan in the interbank market.

Given $l_i(s^i)$ ($i = 1, 2, \cdots, n$), whatever demand for the loan is bank 0's best response for $s^0 = (s_2^a, s_2^b)$ since $t = 0$. In this case, a bank run happens. In other cases of s^0, whatever amount bank 0 chooses, no bank run occurs because at least one agent is a long-lived type (having her dominant strategy of w_3) so that there is no case of both agents choosing withdrawal in period

1. Thus $d > M$ is bank 0's best response for all s^0, taking into account equilibrium behavior of agents at bank 0.

Given $d(s^0) = d > M$ for all s^0, there is always excess demand whatever total supply of loan is provided. This means that $R = \gamma/\alpha$. For $s^i = (s_2^a, s_2^b)$, full default of loan happens in period 3 so that bank i will not lend money to bank 0, i.e., $l_i(s^i)$=0. In this case, even if bank i deviates from this strategy (to a positive amount of loan), agents at bank 0 will not change their strategies due to the conditions in (a) of Proposition 1. This deviation lowers bank i's payoff since its payoff function is $-l_i$ in this case. For $s^i = (s_2^a, s_3^b)$ or $s^i = (s_3^a, s_2^b)$, the short-lived agent fails to pay his debt of the loan but the long-lived agent repays all debt the bank owes so that bank i lends its maximum amount of liquid asset, m_i. Even if bank i deviates (reduces a mount of loan) from this strategy, agents at bank 0 will keep their strategies intact due to condition (viii). This deviation does not increase bank i's payoff. For $s^i = (s_3^a, s_3^b)$, all the loans are surly paid in period 3. It is optimal for bank i to lend m_i to bank 0. Any deviation from this strategy to a smaller amount of loan decreases bank i's payoff since its payoff function is $(\mathrm{R}-1)\,l_i$. Q.E.D.

Under the assumptions of Proposition 2, when a bank run occurs without borrowing, both agents at bank 0 are short-lived. So other banks expect full default of their loans in period 3 when the agents will no longer exist. This makes other banks lend no money to bank 0. In other cases when a bank run does not occur without borrowing, at least one agent is long-lived so that no default will take place, because the long-lived agent has her dominant strategy of w_3 and owes the loans (when the bank is resolved in period 3). This implies that other banks expect high enough expected yields of their loans since the production process without interruption generates a high yield. They lend their liquid assets as much as possible. Banks do not offer loans to a bank at risk of a bank run because a situation where a bank run will happen is associated with a situation where the loans will default.

This linkage *prima facie* appears to correspond to the free market school's view that a solvent bank cannot be illiquid in the well-functioned interbank market. In this view, however, it is supposed that (possibility of) insolvency of a bank induces its depositors to make strategic withdrawals. In our example, it is supposed that even if a lending bank unilaterally makes a loan to a bank in a situation where strategic withdrawals will take place, the lending bank expects a default of its loan due to the linkage to a situation where agents cannot repay the loan they inherit. The causality directs from strategic withdrawals to agents' default of the rescue loan in our example, while in the free market school's view the causality directs from insolvency of a bank to strategic withdrawals (bank runs).

The linkage between the two situations in our example results from the state where both agents are short-lived. However, it is impossible to avoid this state (or possibility) because mismatch of maturity between lending and borrowing is inherent in the modern partial reserve banking system. The interbank market does not necessarily rescue a bank at risk of bank runs by providing liquid assets when the risk of bank runs is linked to a risk of loan default. The bank run is unavoidable since this linkage comes from mismatch of maturity between borrowing and lending in the modern partial reserve banking system.

Ⅳ. Concluding Remarks

This paper offers an example of the game in which banks decide how much to lend or borrow and depositors at a bank at risk of bank runs choose when to withdraw strategically, in order to better understand how the interbank market would (fail to) work to rescue the troubled bank. We modify Postlewaite & Vives (1987)'s example, by adding banks as players and a simple structure of the interbank market. We show two types of equilibrium in our example. There is a Bayesian perfect equilibrium in which the interbank market collapses (with no transaction) and a bank run occurs. There is also an equilibrium where banks refuse to lend and a bank run happens when a situation of a bank run (strategic withdrawal) is linked to a situation of default of the rescue loan. This linkage comes from the fundamental fact of mismatch of maturity between lending and borrowing under the partial reserve banking system. As a result, the interbank market alone does not necessarily save a bank at risk of bank runs.

Several remarks are in order. We have assumed that the borrowing bank (or its manager) tries to minimize a chance of a bank run. This objective of the bank may be justified as a kind of the strategic assumption used when we try to show our result — the interbank market fails to rescue a bank at risk of runs — under the condition — a borrowing bank directly tries to avoid its bank run — that seems disadvantageous to our intention. We can escape from asymmetric treatment between borrowing and lending banks and keep our propositions intact, by assuming that every bank tries to minimize probability of its own bank run. This is a technical trick since the lending banks have no risk of bank runs so that their payoffs are constant, which does not affect equilibrium.

A small amount of liquid assets held by lending banks seems restrictive for our propositions to hold. However, this does not necessarily mean that our results are irrelevant to the real world. If a bank at risk of runs is relatively large to lending banks, the conditions on liquid assets of the lending banks can be satisfied.

A few extensions are worthwhile to examine. We have assumed that the signals received by banks and the pairs of actual type are perfectly correlated. If signals received by lending banks are not informative (not correlated to actual pairs of type) and prior probability of both agents being short-lived is small enough, the lending banks will always lend money to bank 0 in our example (because a bank run occurs only when both agents are short-lived even on off-equilibirum paths and expected payoff of these banks may become positive). On the other hand, if signals received by these banks are not informative and prior probability of both agents being short-lived is not small enough, these banks will not make any loans to bank 0. These cases could be a clue for how a bank's ability to collect information would affect whether the interbank market would rescue a bank at risk of runs.

In the interbank market, banks usually have their specific relations with one another and develop networks among themselves. A bank can receive a rescue loan easily from its closely related banks. We have not taken account of this network structure in our example. As Allen & Gale (2000) demonstrates, the structure of network in the interbank market can be crucial to how the interbank market would function. It is an interesting issue to examine how the structure of network in the interbank market would affect a rescue lending. When lending banks face risk of bank runs, how lending and borrowing through the interbank market are carried out is also a future research topic.

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