Welfare consequence of asymmetric regulation in a mixed Bertrand duopoly

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June 8, 2010

Abstract

I investigate an asymmetric duopoly where a public enterprise must supply the demand it faces, while a private enterprise has no such an obligation. We find that such an asymmetric regulation yields the first best outcome (Walrasian equilibrium).

JEL classification numbers: D43, H44, K23, L51

Key words: supply obligation, unique Bertrand equilibrium, mixed markets
1 Introduction

In mixed oligopoly where one dominant public firm competes against private firms, we often observe asymmetric regulation for the public firm. In Japanese telecommunication market, NTT, which was formerly national firm and still partially held by the Japanese government, must meet the demand it faces, while its private rivals such as KDDI or K-opti.com are not imposed such an obligation. Similar asymmetric obligations are observed in many markets in financial and energy markets. I investigate a welfare consequence of such an asymmetric regulation in a mixed duopoly.¹

I formulate a mixed Bertrand duopoly under strictly convex costs.² I find that the above asymmetric regulation (only public firm must supply the demand it faces) yields the first best outcome (Walrasian equilibrium), whereas the first best outcome fails to be an equilibrium if the regulation is abolished.

Supply obligation has collect broad interests.³ Dastidar (1995) formulated a model where all firms must supply the demand they faces in a private oligopoly and showed that a continuum of equilibria exits.⁴ Ogawa and Kato (2006) investigated the same problem in a mixed duopoly and found that a continuum of equilibria again exits. However, in mixed oligopolies it is not common that both public and private firms are imposed such obligations. Supply obligation is often imposed to public and/or dominant firms only. Thus, I believe that investigating the welfare consequence of the asymmetric regulation is important.⁵

¹ The pioneering work on mixed oligopoly is Merrill and Schneider (1966). They as well as many subsequent works assume that the public firm maximizes social welfare (the sum of consumer’s surplus and profits by firms) while the private firm maximizes its own profits. Recently, studies of mixed markets or mixed oligopoly, involving both private and public enterprises, have become increasingly popular. See Kato and Tomaru (2007) and the works cited by them.

² I can introduce set up costs into the model, which yields U-shape average cost curves. All of the results in the paper hold true as long as firms obtain non-negative profit at the Walrasian equilibrium. Increasing marginal costs and/or U-shape average cost curves are often observed in mixed oligopolies and intensively used in the literature on mixed oligopoly. See Fjell and Pal (1996), Matsumura (1998), and Tomaru and Saito (2010).


⁵ In the literature on mixed oligopolies, public policies such as tax-subsidy, competition, trade, and environmental policies are intensively investigated. See Mujumdar and Pal (1998) for tax policies, White (1996) for subsidy policies, Pal and White (1998) and Mukherjee and Suetrong (2009) for international trade and investment policies, Bárcena-Ruiz and Garzón (2003) for competition policies, Bárcena-Ruiz and Garzón (2006) and Ohori (2006) for environmental policies, Ishibashi and Matsumura for R&D policies, and Matsumura and Kanda (2005) for entry regulation, In spite of the importance, however, the asymmetric supply obligation has been rarely analyzed.
2 The model

I formulate a duopoly model.\footnote{The results of the paper hold in a oligopoly setting. The assumption of duopoly is just for expositional simplicity.} Firm 0 is a welfare-maximizing state-owned public firm and firm 1 is a profit-maximizing private firm. Firms produce perfectly substitutable commodities for which the market demand function is given by \( D(p) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), which is assumed to be continuous and decreasing as long as \( D(p) > 0 \).

All firms have an identical cost function \( C : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), which is assumed to be increasing, continuously differentiable, and strictly convex. We assume \( C(0) = C'(0) = 0 \). In what follows, \( C'(\cdot) \) and its inverse are denoted by \( MC(\cdot) \) and \( Y(\cdot) \), respectively. The profit of firm \( i \) (\( i = 0, 1 \)) is given by \( \Pi_i = p_i q_i - C(q_i) \) and the welfare is given by the sum of consumer surplus and total profits of two firms.

Each firm \( i(i = 0, 1) \) names its price \( p_i \) simultaneously. Firm 0 must meet the demand it faces by the regulation, whereas firm 1 need not. Let \( q_i \) denote firm \( i \)'s output. If \( p_0 < p_1 \), then \( q_0 = D(p_0) \) and \( q_1 = 0 \). If \( p_0 > p_1 \), then \( q_1 = \max\{Y(p_1), D(p_1)\} \) and \( q_0 = \max\{D(p_0) - q_1, 0\} \). If \( p_0 = p_1 \), then \( q_1 = \max\{Y(p_1), D(p_1)/2\} \) and \( q_0 = \max\{D(p_0) - q_1, D(p_0)/2\} \).

3 Results

Before presenting the main result, I present a supplementary result on the equilibrium price.

Lemma 1 \textit{Suppose that only firm 0 has the supply obligation. In equilibrium} \( p_0 > 0 \) \textit{and} \( p_0 \geq p_1 \).

Proof See Appendix.

I now present the main result. Let \( p^W \) be the competitive equilibrium price (Walrasian). It is derived from the market clearing condition \( D(p^W) = 2Y(p^W) \). The following proposition states that the unique equilibrium is Walrasian.

Proposition 1 \textit{Suppose that only firm 0 has the supply obligation. In the unique equilibrium} \( p_0 = p_1 = p^W \).

Proof See Appendix.

Finally, I discuss what happens if the supply obligation is abolished. Suppose that firm 0 need not meet the demand it faces. Let \( q_i \) denote firm \( i \)'s output (\( i, j = 0, 1, i \neq j \)). If \( p_i < p_j \), then \( q_i = \max\{Y(p_i), D(p_i)\} \) and \( q_j = \max\{D(p_j) - q_i, 0\} \). If \( p_0 = p_1 \), then \( q_i = \max\{Y(p_i), D(p_i) - q_j, D(p_i)/2\} \).

Proposition 2 \textit{Suppose that no firm has the supply obligation. There is no equilibrium where} \( p_0 = p_1 = p^W \).
Proof See Appendix.

Propositions 1 and 2 state that asymmetric regulation plays an important role. In the mixed duopoly, the first best is achieved under asymmetric regulation, and not without regulation.\(^7\)

4 Concluding remarks

This paper sheds light on the asymmetric regulation of the supply obligation, which are widely observed in mixed oligopolies. I found that such an asymmetric obligation yields the first best outcome in the unique equilibrium in a mixed duopoly. If the regulation is abolished, this outcome is not an equilibrium outcome. If the regulation is strengthened and symmetric regulation is imposed, this outcome is not an unique equilibrium outcome. Thus, the asymmetric regulation plays an important role for welfare.

Finally, I discuss the robustness of the results. In this paper, I assume that both firms choose their prices simultaneously. In the literature of mixed oligopolies, public and private leaderships and endogenous timing are intensively investigated (See Pal (1998)). Under asymmetric regulation, if the public firm is the Stackelberg leader, the competitive equilibrium again appears. On the other hand, if the private firm is the follower, the equilibrium price is higher than the competitive equilibrium price \(p^W\). If we endogenize the role of the firms by using observable delay game formulated by Hamilton and Slutsky (1990), the unique equilibrium price is \(p^W\). I can show that Propositions 1 and 2 still holds if the timing of price decision is endogenized. In this paper, I assume that the private firm is domestic. In the literature of mixed oligopolies, the nationality of the private firm often affect the equilibrium outcome. Although in this case the competitive equilibrium is not best for domestic welfare, Propositions 1 and 2 still holds even if the private firm is foreign. Thus, I believe that the results of the paper are fairly robust.

Appendix

Proof of Lemma 1 I prove it by contradiction. First I show \(p_0 > 0\) in equilibrium.

Suppose that \(p_0 = p_1 = 0\). Then \(q_1 = 0\) and \(q_0 = D(0)\). Given \(q_1 = 0\), the second best is achieved by \(D(p_0) = Y(p_0)\) (firm 0’s marginal cost is equal to the price). Thus, choosing \(p_0\) such that \(D(p_0) = Y(p_0)\) improves firm 0’s payoff (welfare), a contradiction.

\(^7\) As Dastidar (1997) shows in his model of private oligopoly, if both firms have the obligation, a continuum of equilibria exists. This is true in the mixed oligopoly, too.
Suppose that \( 0 = p_0 < p_1 \). A slight increases of \( p_0 \) reduces \( q_0 \) without changing \( q_1 \). Since \( D(0) > Y(0) \) (price is lower than firm 0’s marginal cost), it improves welfare, a contradiction.

Next, I show that \( p_0 \geq p_1 \). Suppose that \( p_0 < p_1 \). Then \( q_1 = 0 \) and \( \Pi_1 = 0 \). If firm 1 deviates from the equilibrium strategy and chooses an adequate price lower than \( p_0 \), it obtains a positive profit, a contradiction. Q.E.D.

**Proof of Proposition 1** First, I show that \( p_0 = p_1 = p^W \) constitutes an equilibrium.

Given \( p_0 = p^W \), \( q_1 = 0 \) if \( p_1 > p^W \) and \( q_1 = Y(p_1) \) otherwise. The profit of firm 1 is zero if \( p_1 > p^W \), so it is not its best reply. Since \( \Pi_1(p_1) \) is increasing in \( p_1 \) as long as \( q_1 = Y(p_1) \), the optimal price is \( p_1 = p^W \).

Given \( p_1 = p^W \), the first best is achieved when firm 0 chooses \( p_0 = p^W \). Thus it is its best reply.

Next, I show that no other equilibrium exists. I prove it by contradiction. Suppose that \( p_0 = p_1 = p^* > p^W \) in equilibrium. Note that \( q_1 = D(p^*)/2 < Y(p^*) \) since \( D(p^*) < 2Y(p^*) \). If firm 1 deviates from the equilibrium strategy and chooses \( p_1 = p^* - \varepsilon \), \( q_1 = \min\{Y(p^* - \varepsilon), D(p^* - \varepsilon)\} \). Thus, it increases firm 2's profit if \( \varepsilon \) is positive and sufficiently close to 0, a contradiction.

Suppose that \( p_0 = p_1 = p^* < p^W \) in equilibrium. Note that \( 2Y(p^*) > D(p^*) \), \( q_0 > Y(p^*) \), and \( q_1 \leq Y(p^*) \). If firm 0 deviates from the equilibrium strategy and chooses \( p_1 = p' > p^W \) such that \( D(p') = Y(p') + Y(p^*) \), then \( q_1 = Y(p^*) \) and \( q_0 = Y(p') \). This is the second best outcome given \( q_1 \leq Y(p^*) \), so the deviation improves welfare, a contradiction.

Suppose that \( p_0 > p_1 > p^W \). If firm 0 deviates from the equilibrium strategy and chooses \( p_0 = p_1 \), it improves welfare, a contradiction. Suppose that \( p_0 > p_1 = p^* \) and \( p_1 \leq p^W \). Firm 1 can choose \( p'_1 > p^* \) such that \( q_1 = Y(p') \). Since \( \Pi_1(p') \) is increasing in \( p' \) as long as \( q_1 = Y(p') \), the deviation increases the profit of firm 1, a contradiction.

Lemma 1 states that \( p_0 < p_1 \) is impossible in equilibrium. Q.E.D.

**Proof of Proposition 2** Suppose that \( p_0 = p_1 = p^W \). Given \( p_0 = p^W \), \( q_1 = D(p_1) - Y(p^W) \) if \( p_1 \geq p_0 \).

Suppose that firm 1 increases its price slightly. Since

\[
\frac{\partial \Pi_1}{\partial p_1} \big|_{p_1 = p^W} = D(p^W) - Y(p^W) - p^W D' - C'D' = D(p^W) - Y(p^W) = \frac{D(p^W)}{2} > 0,
\]

the deviation increases the profit of firm 1, a contradiction. Note that \( p^W = C' \) when \( p_0 = p_1 = p^W \). Q.E.D.
References


